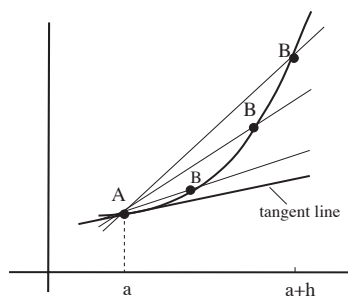
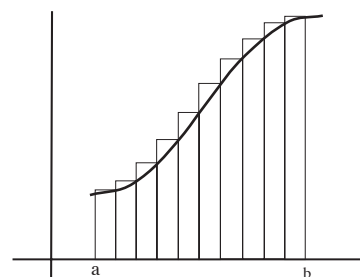


# Sample Questions

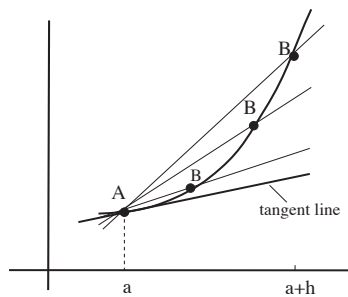
## PREPARING FOR THE AP (BC) CALCULUS EXAMINATION



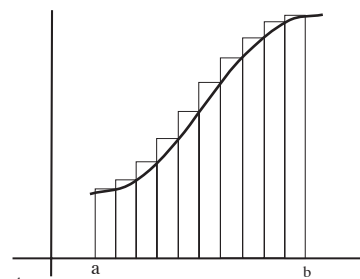
$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$



$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} [f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3 + \dots + f(x_n)\Delta x_n]$$



$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$



$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} [f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3 + \dots + f(x_n)\Delta x_n]$$

Venture Publishing  
9 Bartlet Street  
Andover, MA 01810  
Phone/Fax (508) 896-9486

## Preface

This workbook is intended for students preparing to take the Advanced Placement Calculus BC Examination. It contains six practice tests that are based on the course description published by the College Board. We have tried to make each of the six tests in this workbook as much like the actual AP Exam as possible. For example, in the appropriate sections, there are questions that require students to make decisions about whether to use the graphing calculator a lot, a little, or not at all. In order to provide a greater supply of this type problem, our exams require the use of a calculator in about half the problems of Section I Part B, and all of Section II Part A.

Each student is expected to have a graphing calculator that has the capability to:

- (1) produce the graph of a function within an arbitrary viewing window,
- (2) find the zeros of a function,
- (3) compute the derivative of a function numerically, and
- (4) compute definite integrals.

In the free-response sections, solutions obtained using one of these four capabilities need only show the setup. Solutions using other calculator capabilities must show the mathematical steps that lead to the answer. In either case, a correct answer alone will not receive full credit.

As in the *AP Course Description for Mathematics*, our examinations are in two sections of equal weight. Section I is all multiple-choice and Section II is all free-response.

1. Section I Part A (30 questions in 60 minutes). Calculators may not be used in this part of the exam.
2. Section I Part B (15 questions in 30 minutes). Calculators are allowed.
3. Section II Part A (2 questions in 30 minutes). Calculators are required
4. Section II Part B (4 questions in 60 minutes). Calculators may not be used and the student may go back to Part A if there is time.

We have tried to create the problems in the spirit of *calculus reform*. Calculus reform implies a change in the mode of instruction as well as increased focus on concepts and less attention to symbolic manipulation; emphasis on modeling and applications; use of technology to explore and deepen understanding of concepts; projects and cooperative learning. We have included questions where functions are defined graphically and numerically, as well as symbolically, in order to give the students more practice in this type of analysis.

We wish to thank the members of the Phillips Academy Mathematics Department for their generous contributions of ideas, problems and advice. Their valuable assistance in testing the problems in the classroom has made us quite confident about the validity of the exams.

In the hope of providing future students with a better workbook, the authors welcome your suggestions, corrections, problems of all sorts, and feedback in general. Please send your comments to:

Venture Publishing  
9 Bartlet Street, Suite 55  
Andover, Ma 01810  
Phone/Fax 508-896-9486  
E-Mail gwbest@tiac.net

## Student Preface

There are six examinations in this workbook. Use them as suggested by your teacher, but about two weeks prior to the AP Exam you should try to find a three hour and thirty minute block of time to work through one entire exam. Each part of the exam should be carefully timed. Allow 60 minutes for Section I Part A, 45 minutes for Section I Part B, and 90 minutes for Section II. Take a ten minute break between Part A and Part B and also between Part B and Section II. This will give you a good measure of the topics that need more intensive review as well as give you a feel for the energy and enthusiasm needed on a three hour and fifteen minute exam. Repeat the above routine on a second exam four or five days before the AP to check your progress.

The questions on these exams are designed to be as much like the actual AP Exams as possible. However, we have included a greater percentage of medium level and difficult problems and fewer easy ones, in order to help you gain stamina and endurance. If you do a satisfactory job on these exams, then you should be confident of doing well on the actual AP Exam.

The answers to the multiple-choice questions and selected free-response questions are in the back of the workbook. A complete solution manual for all the problems is available from Venture Publishing. No matter how much of an exam you do at one sitting, we strongly urge you to check your answers when you are finished, not as you go along. You will build your confidence if you DO NOT use the "do a problem, check the answer, do a problem" routine.

The following is a list of common student errors:

1. If  $f'(c) = 0$ , then  $f$  has a local maximum or minimum at  $x = c$ .
2. If  $f''(c) = 0$ , then the graph of  $f$  has an inflection point at  $x = c$ .
3. If  $f'(x) = g'(x)$ , then  $f(x) = g(x)$ .
4.  $\frac{d}{dx} f(y) = f'(y)$
5. Volume by washers is  $\int_a^b (R - r)^2 dx$ .
6. Not expressing answers in correct units when units are given.
7. Not providing adequate justification when justification is requested.
8. Wasting time erasing bad solutions. Simply cross out a bad solution after writing the correct solution.
9. Listing calculator results without the supporting mathematics. Recall that a calculator is to be used primarily to:
  - a) graph functions,
  - b) compute numerical approximations of a derivative and definite integral,
  - c) solve equations.
10. Not answering the question that has been asked. For example, if asked to find the maximum value of a function, do not stop after finding the  $x$ -value where the maximum value occurs.

**EXAM I**  
**CALCULUS BC**  
**SECTION I PART A**  
**Time—60 minutes**  
**Number of questions—30**

**A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION**

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

**In this test:**

- (1) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.
- (2) The inverse of a trigonometric function  $f$  may be indicated using the inverse function notation  $f^{-1}$  or with the prefix “arc” (e.g.,  $\sin^{-1}x = \arcsin x$ ).

1. If  $f'(x) = \ln(x - 2)$ , then the graph of  $y = f(x)$  is decreasing if and only if

- (A)  $2 < x < 3$       (B)  $0 < x$       (C)  $0 < x < 1$       (D)  $x > 1$

Ans

2. Which of the following series is absolutely convergent?

I.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n}$       II.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$       III.  $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{2}\right)^n$

- (A) I only      (B) II only      (C) III only      (D) I and II only

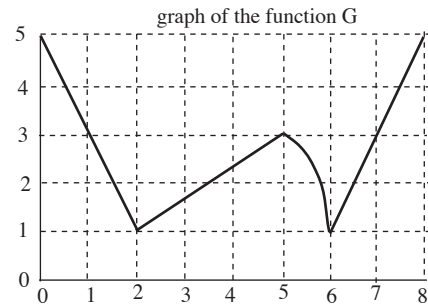
Ans

3. The function  $F$  is defined by

$$F(x) = G[x + G(x)]$$

where the graph of the function  $G$  is shown at the right. The approximate value of  $F'(1)$  is

- (A)  $-2$   
 (B)  $-\frac{2}{3}$   
 (C)  $\frac{2}{3}$   
 (D)  $2$



Ans

4.  $\int_2^6 \left( \frac{1}{x} + 2x \right) dx =$

- (A)  $\ln 4 + 32$   
 (B)  $\ln 3 + 40$   
 (C)  $\ln 3 + 32$   
 (D)  $\ln 4 + 40$

Ans

5. Suppose a population of bears grows according to the logistic differential equation

$$\frac{dP}{dt} = 2P - 0.01P^2$$

where  $P$  is the number of bears at time  $t$  in years. Which of the following are true?

- I. The growth rate of the bear population is greatest at  $P = 100$ .  
 II. If  $P > 200$ , the population of bears is decreasing.  
 III.  $\lim_{t \rightarrow \infty} P(t) = 200$

- (A) I only      (B) II only      (C) I and III only      (D) I, II and III

Ans

6.

$x$	0	4	7	9
$f(x)$	3	$k$	9	11

A function  $f$  is continuous on the closed interval  $[0, 9]$  and has values given in the table above. The trapezoidal approximation for  $\int_0^9 f(t) dt$  found with 3 subintervals, indicated by the data in the table, is 57. What is the value of  $k$ ?

- (A) 9  
 (B) 7  
 (C) 6  
 (D) 5

Ans

7. Which of the following series are conditionally convergent?

I.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$       II.  $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{3^n}$       III.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

- (A) I only      (B) II only      (C) I and II only      (D) I and III only

Ans

8. Which of the following is the solution to the differential equation  $\frac{dy}{dx} = y^2$ , where  $y(-1) = 1$ ?

- (A)  $y = \frac{1}{x}$  for  $x \neq 0$   
 (B)  $y = -\frac{1}{x}$  for  $x < 0$   
 (C)  $y = -\frac{1}{x}$  for  $x > 0$   
 (D)  $y = \frac{1}{x}$  for  $x > 0$

Ans

- 
9.  $\int_1^{\infty} \frac{3x^2}{(1+x^3)^2} dx =$
- (A)  $-\frac{1}{2}$
- (B) 0
- (C)  $\frac{1}{2}$
- (D) 1

Ans

- 
10. Let the first quadrant region enclosed by the graph of  $y = \frac{1}{x}$  and the lines  $x = 1$  and  $x = 4$  be the base of a solid. If cross sections perpendicular to the  $x$ -axis are semicircles, the volume of the solid is
- (A)  $\frac{3\pi}{64}$
- (B)  $\frac{3\pi}{32}$
- (C)  $\frac{3\pi}{16}$
- (D)  $\frac{3\pi}{8}$

Ans

- 
11. The interval of convergence for the series  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$  is
- (A)  $-\infty < x < \infty$
- (B)  $-1 < x < 1$
- (C)  $-1 \leq x < 1$
- (D)  $-1 < x \leq 1$

Ans

**EXAM I**  
**CALCULUS BC**  
**SECTION I PART B**  
**Time—30 minutes**  
**Number of questions—15**

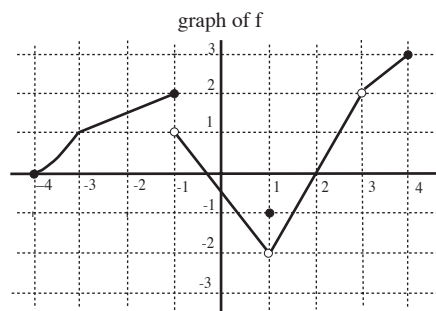
**A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION**

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

**In this test:**

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.
- (3) The inverse of a trigonometric function  $f$  may be indicated using the inverse function notation  $f^{-1}$  or with the prefix “arc” (e.g.,  $\sin^{-1}x = \arcsin x$ ).

1. The function  $f$  is defined on the interval  $[-4, 4]$  and its graph is shown to the right. Which of the following statements are true?



- I.  $\lim_{x \rightarrow 1} f(x) = -1$
- II.  $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = 2$
- III.  $\lim_{x \rightarrow -1^+} f(x) = f(-3)$

- (A) I only      (B) II only      (C) I and II only      (D) II and III only

Ans

2. For  $f(x) = \sin^2 x$  and  $g(x) = 0.5x^2$  on the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , the instantaneous rate of change of  $f$  is greater than the instantaneous rate of change of  $g$  for which value of  $x$ ?

- (A)  $-0.8$       (B)  $0$       (C)  $0.9$       (D)  $1.2$

Ans



3. Which of the following statements is true?

(A) If the limit  $a_n = 0$ , then the series  $\sum a_n$  is convergent.

(B) If  $\sum_{n=1}^{\infty} a_n$  is divergent, then  $\sum_{n=1}^{\infty} |a_n|$  is divergent.

(C) If  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\sum_{n=1}^{\infty} \frac{1}{a_n}$  is convergent

(D) If  $0 \leq a_n \leq b_n$  for all  $n$ , and  $\sum b_n$  diverges, then  $\sum a_n$  diverges.

Ans

4. If  $y^2 - 3x = 7$ , then  $\frac{d^2y}{dx^2} =$

(A)  $\frac{-3}{y^3}$       (B) 3      (C)  $\frac{3}{2y}$       (D)  $\frac{-9}{4y^3}$

Ans

5. The graph of the function  $f$  represented by the Maclaurin series  $1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \frac{16x^4}{4!} + \dots$  intersects the graph of  $y = 2 - x^3$  at the point where  $x =$

(A) 0.182      (B) 0.337      (C) 0.587      (D) 0.799

Ans

6. The rate at which ice is melting in a pond is given by  $\frac{dV}{dt} = \sqrt{1 + 2^t}$ , where  $V$  is the volume of ice in cubic feet, and  $t$  is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 ft<sup>3</sup>      (B) 14.51 ft<sup>3</sup>      (C) 14.53 ft<sup>3</sup>      (D) 14.55 ft<sup>3</sup>

Ans

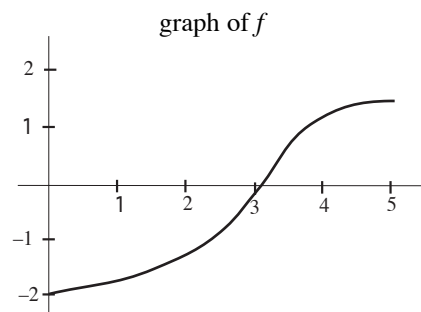
7. Given the differential equation  $\frac{dy}{dx} = x + y$  and  $y(0) = 2$ . An approximation of  $y(1)$  using Euler's method with two steps and step size  $\Delta x = 0.5$  is

(A) 3                      (B)  $\frac{7}{2}$                       (C)  $\frac{15}{4}$                       (D)  $\frac{19}{4}$

Ans

8. The graph of a differentiable function  $f$ , is shown in the figure. If  $h(x) = \int_0^x f(t) dt$ , which of the following are true?

(A)  $h(3) < h'(3) < h''(3)$   
 (B)  $h(3) < h''(3) < h'(3)$   
 (C)  $h'(3) < h(3) < h''(3)$   
 (D)  $h''(3) < h(3) < h'(3)$



Ans

**CALCULUS BC**  
**SECTION II, PART A**  
**Time—30 minutes**  
**Number of problems—2**

**A graphing calculator is required for some problems or parts of problems.**

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.
- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.
- **SHOW ALL YOUR WORK.** Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example,  $\int_1^5 x^2 dx$  may not be written as `fnInt(X2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

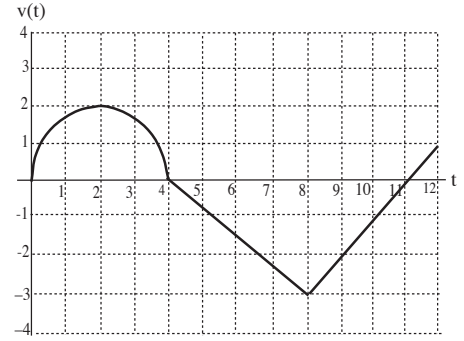
- 
1. A particle moving along a curve in the plane has position  $(x(t), y(t))$  at time where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \quad \text{and} \quad \frac{dy}{dt} = e^t + 5e^{-t}$$

for all real values  $t$ . At time  $t = 0$ , the particle is at the point  $(4, 1)$ .

- (a) Find the speed of the particle and its acceleration at time  $t = 0$ .
- (b) Find an equation of the line tangent to the path of the particle at time  $t = 0$ .
- (c) Find the x-coordinate of the position of the particle at time  $t = 3$ .
-

2. A particle moves along the  $x$ -axis so that its velocity at time  $0 \leq t \leq 12$ , is given by a differentiable function  $v$  whose graph is shown at the right. The velocity graph consists of a semicircle and two line segments.



- (a) At what time,  $0 \leq t \leq 12$ , is the speed of the particle the greatest?
- (b) At what times, if any, does the particle change direction? Explain briefly.
- (c) Find the total distance traveled by the particles from time  $t = 0$  to  $t = 12$ .
- (d) If the initial position of the particle is  $x(0)=2$ , find the position of the particle at time  $t=8$ .

---

**Time - 60 minutes**  
**Number of problems - 4**

**A graphing calculator may NOT be used on this part of the examination.**

- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

---

3. Let  $f$  be a function with derivatives of all orders and for which  $f(2) = 0$ . The  $n^{\text{th}}$  derivative of  $f$  at  $x = 2$  is given by  $f^n(2) = \frac{n!}{n \cdot 3^n}$ .

- (a) Write the third-degree Taylor polynomial for  $f$  about  $x = 2$ .
- (b) Find the interval of convergence of the Taylor series for  $f$  about  $x = 2$ . Show the work that leads to your answer.
-

**Answers****Multiple Choice Part A**

- |    |   |
|----|---|
| 1. | C |
| 2. | C |
| 3. | B |
| 4. | C |
| 5. | D |
| 6. | D |
| 7. | D |
| 8. | B |
| 9. | C |
| 10 | B |
| 11 | C |

**Multiple Choice Part B**

- |    |   |
|----|---|
| 1. | D |
| 2. | C |
| 3. | B |
| 4. | D |
| 5. | B |
| 6. | C |
| 7. | D |
| 8. | A |

**Free Response PART A**

- |    |                                                                      |
|----|----------------------------------------------------------------------|
| 1. | a) speed= $\sqrt{58}$ ; accelerator vector = $\langle 0, -3 \rangle$ |
|    | b) $\frac{dy}{dx} = \frac{7}{3}$ ; $y = \frac{7}{3}(x - 4) + 1$      |
|    | c) 17.930                                                            |
| 2. | a) $t = 8$ b) $t = 4$ and $t = 11$                                   |
|    | c) $2\pi + 11$ d) $2\pi - 4$                                         |

**Free Response PART B**

- |    |                                                                     |
|----|---------------------------------------------------------------------|
| 3. | a) $0.4 \text{ ft} / \text{sec}^2$ b) 1300 ft    c) car A is faster |
|----|---------------------------------------------------------------------|