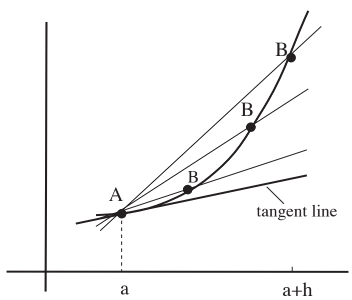
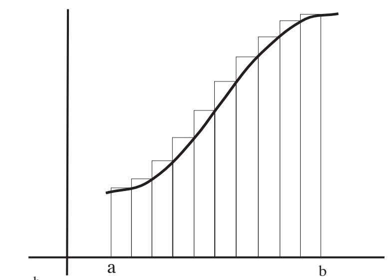
Sample Questions

PREPARING FOR THE

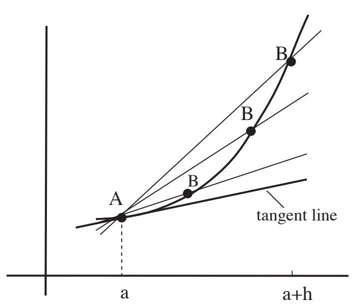
AP (AB) CALCULUS EXAMINATION



$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$



$\int_a^b f(x)\,dx = \lim_{n \to \infty} [f(x\,)\Delta x_1 + f(x\,)\Delta x_2 + f(x\,)\Delta x_3 + ... + f(x\,)\Delta x_n]$



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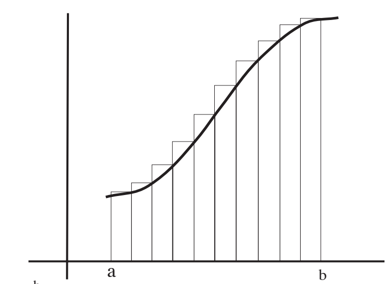

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Preface

This workbook is intended for students preparing to take the Advanced Placement Calculus AB Examination. It contains six practice tests that are based on the course description published by the College Board. We have tried to make each of the six tests in this workbook as much like the actual AP Exam as possible. For example, in the appropriate sections, there are questions that require students to make decisions about whether to use the graphing calculator a lot, a little, or not at all. In order to provide a greater supply of this type problem, our exams require the use of a calculator in about half the problems of Section I Part B, and all of Section II Part A.

Each student is expected to have a graphing calculator that has the capability to:

(1) produce the graph of a function within an arbitrary viewing window,
(2) find the zeros of a function,
(3) compute the derivative of a function numerically, and
(4) compute definite integrals.

In the free-response sections, solutions obtained using one of these four capabilities need only show the setup. Solutions using other calculator capabilities must show the mathematical steps that lead to the answer. In either case, a correct answer alone will not receive full credit.

As in the *AP Course Description for Mathematics*, our examinations are in two sections of equal weight. Section I is all multiple-choice and Section II is all free-response.

1. Section I Part A (30 questions in 60 minutes). Calculators may not be used in this part of the exam.
2. Section I Part B (15 questions in 45 minutes). Calculators are allowed.
3. Section II Part A (2 questions in 30 minutes). Calculators are required
4. Section II Part B (4 questions in 60 minutes). Calculators may not be used and the student may go back to Part A if there is time.

We have tried to create the problems in the spirit of *calculus reform*. Calculus reform implies a change in the mode of instruction as well as increased focus on concepts and less attention to symbolic manipulation; emphasis on modeling and applications; use of technology to explore and deepen understanding of concepts; projects and cooperative learning. We have included questions where functions are defined graphically and numerically, as well as symbolically, in order to give the students more practice in this type of analysis.

We wish to thank the members of the Phillips Academy Mathematics Department for their generous contributions of ideas, problems and advice. Their valuable assistance in testing the problems in the classroom has made us quite confident about the validity of the exams. Robert Clements of Phillips Exeter Academy provided excellent editorial assistance and insightful comments.

In the hope of providing future students with a better workbook, the authors welcome your suggestions, corrections, problems of all sorts, and feedback in general. Please send your comments to:

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Student Preface

There are six examinations in this workbook. Use them as suggested by your teacher, but about two weeks prior to the AP Exam you should try to find a three hour and thirty minute block of time to work through one entire exam. Each part of the exam should be carefully timed. Allow 60 minutes for Section I Part A, 45 minutes for Section I Part B, and 90 minutes for Section II. Take a ten minute break between Part A and Part B and also between Part B and Section II. This will give you a good measure of the topics that need more intensive review as well as give you a feel for the energy and enthusiasm needed on a three hour and fifteen minute exam. Repeat the above routine on a second exam four or five days before the AP to check your progress.

The questions on these exams are designed to be as much like the actual AP Exams as possible. However, we have included a greater percentage of medium level and difficult problems and fewer easy ones, in order to help you gain stamina and endurance. If you do a satisfactory job on these exams, then you should be confident of doing well on the actual AP Exam.

The answers to the multiple-choice questions and selected free-response questions are in the back of the workbook. A complete solution manual for all the problems is available from Venture Publishing. No matter how much of an exam you do at one sitting, we strongly urge you to check your answers when you are finished, not as you go along. You will build your confidence if you DO NOT use the "do a problem, check the answer, do a problem" routine.

The following is a list of common student errors:

1. If $f'(c) = 0$, then f has a local maximum or minimum at $x = c$.
2. If $f''(c) = 0$, then the graph of f has an inflection point at $x = c$.
3. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
4. $\frac{d}{dx} f(y) = f'(y)$
5. Volume by washers is $\int_a^b (R - r)^2 dx$.
6. Not expressing answers in correct units when units are given.
7. Not providing adequate justification when justification is requested.
8. Wasting time erasing bad solutions. Simply cross out a bad solution after writing the correct solution.
9. Listing calculator results without the supporting mathematics. Recall that a calculator is to be used primarily to:
   a) graph functions,
   b) compute numerical approximations of a derivative and definite integral,
   c) solve equations.
10. Not answering the question that has been asked. For example, if asked to find the maximum value of a function, do not stop after finding the x-value where the maximum value occurs.

EXAM I
CALCULUS AB
SECTION I PART A
Time–60 minutes
Number of questions–30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. If $f'(x) = \frac{1}{1+x^2}$ and $g'(x) = \frac{1}{\left(1+\frac{x^2}{4}\right)} \cdot \frac{1}{2}$ for all x, and if $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$,

then $\lim_{x\to 0} \frac{f(x)}{g(x)} =$

(A) –8 (B) –2 (C) $\frac{1}{2}$ (D) 2

Ans ☐

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2. For $x \neq 0$, the slope of the tangent to the graph of $y = x\cos x$ equals zero whenever

(A) $\tan x = -x$

(B) $\tan x = \frac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

Ans ☐

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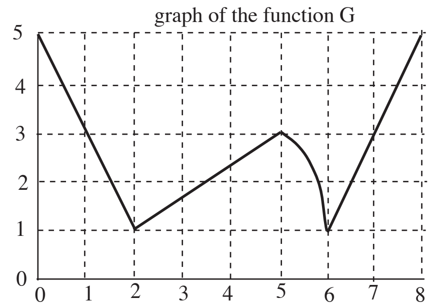
3. The function F is defined by

$F(x) = G[x + G(x)]$

where the graph of the function G is shown at the right.



The approximate value of $F'(1)$ is

(A) $\frac{7}{3}$

(B) $\frac{2}{3}$

(C) -1

(D) $-\frac{2}{3}$

Ans

4. $\int_2^6 \left(\frac{1}{x} + 2x\right)\, dx =$

(A) ln 4 + 32

(B) ln 3 + 40

(C) ln 3 + 32

(D) ln 4 + 40

Ans

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs at

(A) 1

(B) 2

(C) e

(D) e^2

Ans

6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral $\int_{-1}^{3} |2x - 3| \, dx$.

(A) 13

(B) 10

(C) 8.5

(D) 8

Ans

7. An equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -3x + 1$

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

Ans

8. $\int \cos(3 - 2x) \, dx =$

(A) $\sin(3 - 2x) + C$

(B) $-\sin(3 - 2x) + C$

(C) $\frac{1}{2}\sin(3 - 2x) + C$

(D) $-\frac{1}{2}\sin(3 - 2x) + C$

Ans

9. Propane is pumped into a tank at a constant rate of 6 gallons per minute. Propane leaks out of the tank at the rate of $\frac{1}{\sqrt{t+1}}$ gallons per minute. At $t = 0$ the tank contains 20 gallons of propane. How many gallons of propane are in the tank at $t = 8$ minutes?

(A) 60

(B) 64

(C) 68

(D) 72

Ans

10. Let the first quadrant region enclosed by the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are semicircles, the volume of the solid is

(A) $\frac{3\pi}{64}$

(B) $\frac{3\pi}{32}$

(C) $\frac{3\pi}{16}$

(D) $\frac{3\pi}{8}$

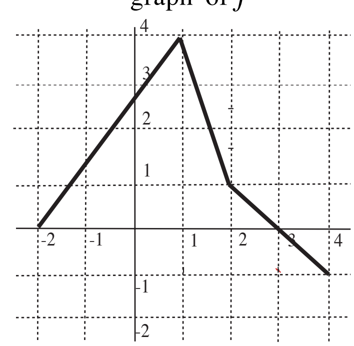
Ans

11. The graph of the function f, consisting of three line segments, is shown in the figure. Let

$$g(x) = \int_{-2}^{x} f(t)\, dt.$$

Then $g''(0) =$

graph of f



(A) $\frac{3}{2}$

(B) $\frac{4}{3}$

(C) 0

(D) -3

Ans

EXAM I
CALCULUS AB
SECTION I PART B
Time–45 minutes
Number of questions–15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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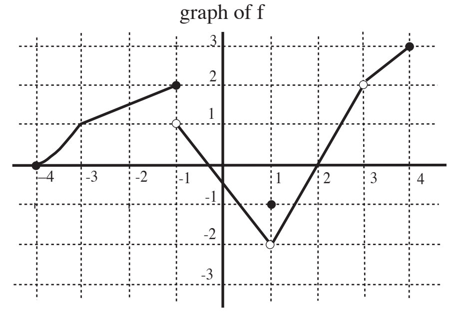
1. The function f is defined on the interval [–4, 4] and its graph is shown to the right. Which of the following statements are true?



I. $\lim_{x \to 1} f(x) = -1$

II. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 2$

III. $\lim_{x \to -1^+} f(x) = f(-3)$

(A) I only (B) II only (C) II and III only (D) I, II, III

Ans

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2. Find the average value of the acceleration of a particle whose velocity is modeled by the function $v(t) = t + 2\sin t$ on the interval [0, 2].

(A) 0.604 (B) 1.205 (C) 1.910 (D) 2.416

Ans

3.

x	-1	1	3	4
$f(x)$	3	1	3	2

Let f be a polynomial function with values at selected inputs recorded in the table above. Which of the following must be true for $-1 < x < 4$?

(A) The graph of f is concave up.

(B) The graph of f has at least two points of inflection.

(C) f is increasing.

(D) f has at least two relative extrema.

Ans

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4. If $y^2 - 3x = 7$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{-6}{7y^3}$ (B) $\dfrac{-3}{y^3}$ (C) 3 (D) $\dfrac{-9}{4y^3}$

Ans

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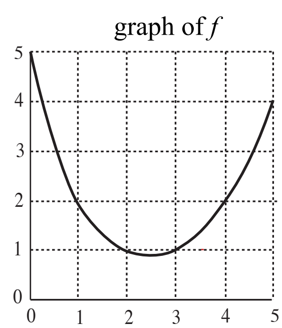
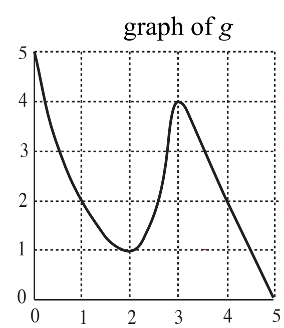
5. The graphs of functions f and g are shown at the right.
If $h(x) = g[f(x)]$, which of the following statements are true about the function h?





I. $h(0) = 4$.

II. h is increasing at $x = 2$.

III. The graph of h has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) II and III only (D) I, II, III

Ans

6. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1 + 2^t}$, where V is the volume of ice in cubic feet, and t is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 ft^3 (B) 14.51 ft^3 (C) 14.53 ft^3 (D) 14.55 ft^3

Ans

7. The region shaded in the figure at the right is rotated about the x-axis. Using the Trapezoid Rule with 5 equal subdivisions, the approximate volume of the resulting solid is

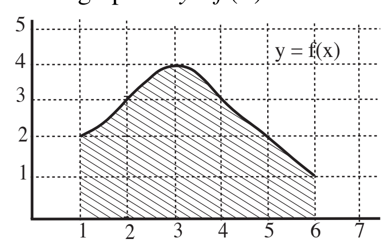
(A) 23

(B) 47

(C) 127

(D) 254

graph of $y = f(x)$



Ans

8. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y(-2) = -1$?

(A) $y = -\sqrt{x^2 - 3}$ for $x > \sqrt{3}$

(B) $y = \sqrt{x^2 - 3}$ for $x > \sqrt{3}$

(C) $y = \sqrt{x^2 - 3}$ for $x < -\sqrt{3}$

(D) $y = -\sqrt{x^2 - 3}$ for $x < -\sqrt{3}$

Ans

CALCULUS AB
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. The position of a particle moving on the x-axis at time $t > 0$ seconds is: $x(t) = e^t - \sqrt{t}$ feet.

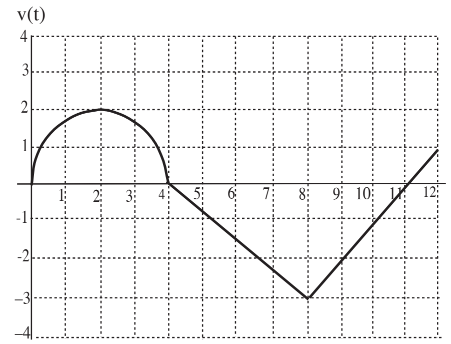
   (a) Find the average velocity of the particle over the interval $1 \le t \le 3$.

   (b) In what direction and how fast is the particle moving at $t = 1$ seconds?

   (c) For what values of t is the particle moving to the right?

   (d) Find the position of the particle when its velocity is zero.

2. A particle moves along the x-axis so that its velocity at time $0 \leq t \leq 12$, is given by a differentiable function v whose graph is shown at the right. The velocity graph consists of a semicircle and two line segments.



(a) At what time. $0 \leq t \leq 12$, is the speed of the particle the greatest?

(b) At what times, if any, does the particle change direction? Explain briefly.

(c) Find the total distance traveled by the particls from time $t = 0$ to $t = 12$.

(d) If the initial position of the particle is $x(0)=2$, find the position of the particle at time $t = 8$.

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

3. Car A has positive velocity $v(t)$ as it travels along a straight road, where v is a differentiable function of t. The velocity of the car is recorded for several selected values of t over the interval $0 \le t \le 60$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60
$v(t)$ (feet per second)	5	14	7	11	12	40	44

(a) Use the data from the table to approximate the acceleration of Car A at $t = 25$ seconds. Show the computation that lead to your answer. Indicate units of measure.

(b) Use the data from the table to approximate the distance traveled by Car A over the time interval $0 \le t \le 60$ seconds by using a midpoint Riemann sum with 3 subdivisions of equal length. Show the work that lead to your answer.

(c) Car B travels along the same road with an acceleration of $a(t) = \dfrac{1}{\sqrt{x+9}}$ ft / sec^2.
At time $t = 0$ seconds, the velocity of Car B is 3 ft/sec. Which car is traveling faster at $t = 40$ seconds? Show the work that lead to your answer

Multiple Choice Part A

1.	D
2.	B
3.	D
4.	C
5.	D
6.	D
7.	A
8.	D
9.	B
10	B
11	B

Multiple Choice Part B

1.	D
2.	C
3.	D
4.	D
5.	C
6.	C
7.	C
8.	D

Free Response PART A

1. a) 8.318

   b) $v(1) = e - \frac{1}{2} > 0$ so it moves to the right at $e - \frac{1}{2} \approx 2.218$ ft/sec.

   c) $t > 0.176$ d) $x(.1756) = 0.773$

2. a) t = 8 b) t = 4 and t =11
   c) 2π+11 d) 2π – 4

Free Response PART B

3. a) $0.4\,ft / \sec^2$ b) 1300 ft c) car A is faster